K17U 2677

Reg. No. : ....

Name : ....

**I Semester B.Sc. Hon's (Mathematics) Degree (Regular/Supple./Improv.)
Examination, November 2017
BHM 102 : FOUNDATIONS OF MATHEMATICS
(2016 Admission)**

Time : 3 Hours

Max. Marks : 60

SECTION – A

(Answer **any 4** questions out of 5 questions. **Each** question carries **1** mark.)   **(4×1=4)**

1. Define a bijective function.

2. Define De-Morgan's laws.

3. Define a countably infinite set.

4. State well ordering theorem for a set.

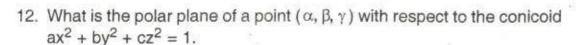5. What is the locus of the equation $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} + \dfrac{z^2}{c^2} = 1$.

SECTION – B

(Answer **any 6** questions out of 9 questions. **Each** question carries **2** marks.)   **(6×2=12)**

6. Define the restriction of a function f and give an example.

7. Define the least upper bound and greatest lower bound of a set.

8. Distinguish between the conjunction and disjunction of a statement P.

9. How many normals can be drawn to a paraboloid from a given point.

10. Define the angle of intersection of two spheres.

11. True or false : Cone is a central conicoid.

P.T.O.

12. What is the polar plane of a point (α, β, γ) with respect to the conicoid
$ax^2 + by^2 + cz^2 = 1$.

13. Define contrapositive of a statement. Give an example.

14. Define an equivalence relation.

SECTION – C

(Answer **any 8** questions out of 12 questions. **Each** question carries **4** marks.) **(8×4=32)**

15. Given $f : R \to R$ by $f(x) = 10x^2 + 6$ and $g : R \to R$ by $g(x) = 9x + 5$. Find $f_{\circ}g$ and $g_{\circ}f$.

16. Find the least upper bound of the following sets.

   1) $\{-1/2n : n \in N\}$

   2) $\left\{\dfrac{1}{n} : n \in N\right\}$.

17. Show that countable union of countable sets is countable.

18. Show that the square of an odd integer is an odd integer.

19. If m, n are natural numbers such that $m + n \geq 20$, then show that either $m \geq 10$ or $n \geq 10$.

20. Show that two equivalent classes E and E′ are either disjoint or equal.

21. Find the centre and radius of the sphere $x^2 + y^2 + z^2 + 2x - 4y - 6z + 5 = 0$.

22. Find equation of the right circular cylinder with radius 2 and axis is given by
$\dfrac{x-1}{2} = \dfrac{y}{3} = \dfrac{z-3}{1}$.

23. Show that the plane $lx + my + nz = p$ will touch the sphere
$x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$ if $(ul + vm + wn + p)^2 = (l^2 + m^2 + n^2)(u^2 + v^2 + w^2 - d)$.

24. Find the equation of the sphere passing through the circle.
$x^2 + y^2 + z^2 - 2x + 3y - 4z + 6 = 0$, $3x - 4y + 5z - 15 = 0$ and cuts the sphere
$x^2 + y^2 + z^2 + 2x + 4y - 6z + 11 = 0$ orthogonally.

25. Show that the semi vertical angle of a right circular cone having sets of three mutually perpendicular generators is $\tan^{-1}(\sqrt{2})$.

26. Show that the sum of squares of three conjugate semi diameters of an ellipsoid is constant.

SECTION – D

(Answer **any 2** questions out of 4 questions. **Each** carries **6** marks.) **(2×6=12)**

27. Define section of a well ordered set X by a. If A is a countable subset of S_{Ω} then show that A has an upper bound in S_{Ω}.

28. Find the equation of a sphere passing through four points $(0, 0, 0)$, $(-a, b, c)$, $(a, -b, c)$ and $(a, b, -c)$.

29. Find the two tangent planes to the sphere $x^2 + y^2 + z^2 - 4x + 2y - 6z + 5 = 0$, which are parallel to the plane $2x + 2y = z$.

30. What are the equations relating spherical coordinates of a point to Cartesian and cylindrical coordinates. Also find a spherical coordinate equation for the cone
$z = \sqrt{x^2 + y^2}$.

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