K16U 2596

Reg. No. : .....

Name : .....

I Semester B.Sc. (Hon's) (Mathematics) Degree (Reg./Supple./Improve.) Examination, November 2016

BHM – 102 : FOUNDATIONS OF MATHEMATICS

Time : 3 Hours | Max. Marks : 80

Answer **all** the **ten** questions : **(10×1=10)**

1. If $A \setminus B = \phi$ then $A \cup B = ?$
2. Let V = {1, 2, 3, 4}. Is f_1 = {(2, 3), (1, 4), (2, 1), (3, 2), (4, 4)} a function from V into V.
3. Let W = {1, 2, 3, 4} and R = {(1, 1), (1, 3), (2, 2), (3, 1), (4, 4)}. Is R reflexive ?
4. Give an example of a partial ordering relation on the set of positive integers.
5. When does a constant function become One-to-One ?
6. If $B_i = [i, i + 1]$, $i \in Z$, the set of integers, find $\cup (B_i : i \in Z)$.
7. Find x and y given (2x, x + y) = (6, 2).
8. Define the term "Proposition".
9. If Q(x, y) denote the statement x = y + 3 what is the truth value of Q(1, 2) ?
10. When do you say that two sets A and B are equipotent ?

Answer **any 10** short answer questions out of 14 : **(10×3=30)**

11. Suppose A = {a, b} and B = {1, 2, 3}. Find the number of functions from A into B.
12. Let $f : R \to R$ be defined by $f(x) = x^2 + 2x$. Find f o f (x).

P.T.O.

13. Consider the function $f : R \to R$ defined by $f(x) = x^2$. Find $f^{-1}(\{x : x \leq 0\})$.

14. Give an example of a function from N to N that is :
    i) One-to-One but not Onto
    ii) Onto but not One-to-One.

15. Let f be a function from set A to set B and let S and T be sub-sets of A. Show that $f(S \cup T) = f(S) \cup f(T)$.

16. If f and f o g are One-to-One does it follow that g is One-to-One ? Justify your answer.

17. Suppose A is a non-empty set and f is a function that has A as its domain. Let R be the relation on A consisting of all ordered pairs (x, y) where f(x) = f(y). Show that R is an equivalence relation.

18. Show that the propositions $p \vee (q \wedge r)$ and $(p \vee q) \wedge (p \vee r)$ are logically equivalent.

19. What is the truth value of $\exists x P(x)$ where P(x) is the statement "$x^2 > 10$" and the universe of discourse consists of positive integers not exceeding 4 ?

20. Explain "Valid Argument". Give an example.

21. Prove that $(A \times B) \cap (A \times C) = A \times (B \cap C)$.

22. Form a rational cubic equation whose two roots are 2 and 3 + i.

23. Prove that a sub-set of a denumerable set is finite or denumerable.

24. If $\alpha, \beta, \gamma, \delta$ are the roots of $x^4 + ax^3 + bx^2 + cx + d = 0$ find the value of $\sum \alpha^2$.

Answer **any 6** short answer questions out of 9 : **(6×5=30)**

25. Let A be a set of non-zero integers and let $\equiv$ be a relation on A × A defined as follows :
    (a, b) $\equiv$ (c, d) whenever ad = bc.
    Prove that $\equiv$ is an equivalence relation.

26. Prove that a countable union of finite sets is countable.

27. Prove that $(0, 1] \approx [0, 1]$.

28. Show that $(p \wedge q) \to (p \vee q)$ is a tautology.

29. Solve the equation $x^4 - 10x^3 - 120x^2 + 320x + 1024 = 0$ given that the roots are real and they are in G.P.

30. If α, β, γ are the roots of $x^3 + px^2 + qx + r = 0$, prove that $(\alpha + \beta)(\beta + \gamma)(\alpha + \gamma) = r - pq$.

31. Solve $x^4 + 3x^3 - 3x - 1 = 0$.

32. Show that the statement every "positive integer is the sum of the squares of three integers is false".

33. i) Express the definition of a limit using quantifiers.
    ii) Translate the statement "The sum of two positive integers is positive" in to a logical expression.

Answer **any one** essay question out of 2 : **(1×10=10)**

34. i) Solve the equation $x^4 + 2x^3 - x^2 - 2x - 3 = 0$
    ii) Solve $x^3 - 6x^2 + 3x - 2 = 0$.

35. Show that :
    i) $(\neg P \wedge (\neg Q \wedge R)) \vee (Q \wedge R) \vee (P \wedge R) \equiv R$
    ii) $((P \vee Q) \wedge \neg (\neg P \wedge (\neg Q \vee \neg R))) \vee (\neg P \wedge \neg Q) \vee (\neg P \wedge \neg R)$ is a tautology.
