Reg. No. : ....

Name : ....

I Semester B.Sc. Hon's (Mathematics) Degree (Regular) Examination, November 2016
BHM 102 : FOUNDATIONS OF MATHEMATICS
(2016 Admission)

Time : 3 Hours | Max. Marks : 60

SECTION – A

Answer any 4 questions out of 5 questions. Each question carries 1 mark : (4×1=4)

1. Define a function and give an example.
2. Define a countable set and give an example.
3. How many normals can be drawn to a central quadric ?
4. Define the angle of intersection of two spheres.
5. Define the supremum and infimum of two sets.

SECTION – B

Answer any 6 questions out of 9 questions. Each question carries 2 marks : (6×2=12)

6. Given $f : R \to R$ defined by $f(x) = 3x^2 + 2$ and $g : R \to R$ defined by $g(x) = 5x$. Find gof and fog.
7. Give an example of function which is :
   a) neither injective nor surjective
   b) injective but not surjective.
8. If m, n are natural numbers such that $m + n \geq 20$, then show that either $m \geq 10$ or $n \geq 10$.

P.T.O.

9. Find the intersection of a sphere $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$ and a line $\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n}$.

10. Find the principal planes and principal axis of the hyperboloid of 1 sheet and hyperboloid of 2 sheets.

11. Define polar plane of a point and a pole.

12. Find the equation of the tangent plane of the sphere at (α, β, γ).

13. Show that the semivertical angle of a right circular cone having sets of three mutually perpendicular generation is $\tan^{-1}\sqrt{2}$.

14. Define an order relation. Give an example.

SECTION – C

Answer **any 8** questions out of **12** questions. **Each** question carries **4** marks : **(8×4=32)**

15. Define the restriction of a function. Explain with an example.

16. Show that countable union of a countable set is countable.

17. Define section of a well ordered set Y by a. If A is a countable subset of S_Ω then show that A has an upper bound in S_Ω.

18. Show that the square of an odd integer is also an odd integer.

19. Find the equation of a sphere passing through the four points $(4, -1, 2)$, $(0, -2, 3)$, $(1, -5, -1)$ and $(2, 0, 1)$.

20. Show that the plane $lx + my + nz = p$ will touch the sphere $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$ if $(ul + vm + wn + p)^2 = (l^2 + m^2 + n^2)(u^2 + v^2 + w^2 - d)$.

21. Find the points of intersection of the line $-\frac{1}{3}(x+5) = y-4 = \frac{1}{7}(z-11)$ with the central conicoid $12x^2 - 7y^2 + 7z^2 = 7$.

22. Show that the sum of the squares of three conjugate semidiameters is constant.

23. Find the enveloping cone of the conicoid $ax^2 + by^2 + cz^2 = 1$ with vertex (α, β, γ).

24. Show that a finite product of countable sets is countable.

25. Show that $4x^2 - y^2 + 2z^2 + 2xy - 3yz + 12x - 11y + 6z + 4 = 0$ represents a cone with vertex $(-1, -2, -3)$.

26. Find the point of intersection of a sphere $x^2 + y^2 + z^2 + 2x - 10y = 23$ and a line $\frac{1}{4}(x+3) = \frac{1}{3}(y+4) = \frac{-1}{5}(z-8)$.

SECTION – D

Answer **any 2** questions out of **4** questions. **Each** question carries **6** marks : **(2×6=12)**

27. Let B be a non-empty set. Show that B is countable $\Leftrightarrow$ there is a surjective function $f: Z_+ \to B \Leftrightarrow$ there is an injective function $g: B \to Z_+$.

28. Find the equations of two tangent planes to the sphere $x^2 + y^2 + z^2 = 9$ which passes through the lines $x + y = 6$, $x - 2z = 3$.

29. Discuss the nature and geometrical properties of the Elliptic paraboloid and hyperbolic paraboloid.

30. Find the two tangent planes to the sphere $x^2 + y^2 + z^2 - 4x + 2y - 6z + 5 = 0$ which are parallel to the plane $2x + 2y = z$.
