K18U 2250

Reg. No. : .....

Name : .....

I Semester B.Sc. Hon's (Mathematics) Degree (Reg./Supple./Improv.) Examination, November 2018
(2016 Admn. Onwards)
BHM 103 : DIFFERENTIAL CALCULUS

Time : 3 Hours Max. Marks: 60

SECTION – A

Answer **any 4** questions out of 5 questions. **Each** question carries **1** mark.

1. What is the value of $\lim_{x\to 2} \frac{x+3}{x+6}$?
2. Find the derivative of $(3 - x^2)(x^3 - x + 1)$.
3. What is the n^{th} derivative of $\log(ax + b)$?
4. State true or false : As $x \to \infty$, $x + \sin x = O(x)$.
5. If $u = \frac{2y}{y+\cos x}$, what is the value of $\frac{\partial u}{\partial x}$? **(4×1=4)**

SECTION – B

Answer **any 6** questions out of 9 questions. **Each** question carries **2** marks.

6. What do you mean by infinite limits ?
7. State max-min theorem for continuous functions.
8. State Rolle's theorem and mean value theorem
9. Discuss the concavity of $y = x^3$.

P.T.O.

10. Find the n^{th} derivative of $\frac{x+1}{x^2-4}$.

11. Define exponential function. Write the law of exponents.

12. Evaluate $\int \frac{\log_2 x}{x} dx$.

13. Show that $\lim_{(x,y)\to(0,0)} \frac{2x^2y}{x^4+y^2}$ does not exist.

14. Find $\frac{dw}{dt}$ if $w = xy + z$, $x = \cos t$, $y = \sin t$, $z = t$. **(6×2=12)**

SECTION – C

Answer **any 8** questions out of 12 questions. **Each** question carries **4** marks.

15. State Sandwitch theorem. Use this theorem to find $\lim_{x\to 0} u(x)$ if

$1 - \frac{x^2}{4} \le u(x) \le 1 + \frac{x^2}{2}$ for all $x \neq 0$.

16. If $\lim_{x\to c} f(x) = L$ and $\lim_{x\to c} g(x) = M$ prove that $\lim_{x\to c} (f(x) + g(x)) = L + M$.

17. What do you mean by left hand limit and right hand limit ? Show that $\sin(1/x)$ has no limit as x approaches zero from either side.

18. Explain the idea of continuity of a function at a point. Discuss the continuity of $f(x) = |x|$.

19. Find two positive numbers whose sum is 20 and whose product is as large as possible.

20. Find the n^{th} derivative of $(2x + 3)^3 \log (2x + 1)$.

21. Find $\frac{dy}{dx}$ if $y = \frac{(x^2+1)(x+3)^{1/2}}{x-1}$, $x > 1$.

22. Find $\lim_{x\to 0^+} + (1 + x)^{1/x}$.

23. What do you mean by inverse hyperbolic functions ? Show that if u is a differentiable function of x whose values are greater than 1, then

$\frac{d}{dx}(\cosh^{-1}(u)) = \frac{1}{\sqrt{u^2-1}} \frac{du}{dx}$.

24. Find the linearization of $f(x, y, z) = x^2 - xy + 3 \sin z$ at the point (2, 1, 0).

25. Find $\frac{\partial w}{\partial x}$ at the point (2, – 1, 1) if $w = x^2 + y^2 + z^2$, $z^3 - xy + yz + y^3 = 1$, and x and y are the independent variables.

26. Find the local extreme values of the function $f(x, y, z) = x^2 + xy + y^2 + 3x - 3y + 4$. **(8×4=32)**

SECTION – D

Answer **any 2** questions out of 4 questions. **Each** question carries **6** marks.

27. Find the critical points of $f(x) = x^{1/3}(x - 4)$. Identify the intervals on which f is increasing and decreasing. Find the function's local and absolute extreme values.

28. If $y = \sin(m \sin^{-1} x)$, prove that $(1 - x^2) y_{n+2} - (2n + 1)xy_{n+1} + (m^2 - n^2)y_n = 0$.

29. a) State and prove the first form of L'Hospital rule for indeterminate forms.

b) Evaluate $\lim_{x\to 0} \left(\frac{1}{\sin x} - \frac{1}{x}\right)$.

30. The plane $x + y + z = 1$ cuts the cylinder $x^2 + y^2 = 1$ in an ellipse. Find the points on the ellipse that lie closest to and farthest from the origin. **(2×6=12)**
